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Valeria Saggio

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## Experimental few-copy multi-particle entanglement detection

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V. Saggio, A. Dimić, C. Greganti, L. A. Rozema, P. Walther, and B. Dakić

*arXiv:1809.05455 [quant-ph]*

# Outline

- **Motivation;**
- **Theoretical background:** entanglement detection as a probabilistic procedure;
  - Detecting entanglement in cluster states.
- **Experiment:** generation of a six-qubit cluster state at telecommunication wavelengths;
- **Results:** entanglement verified in the state with high confidence;
- **Conclusions:** outlooks and perspectives.

# Motivation

- Reliable verification of quantum entanglement is a considerable challenge when dealing with large-scale quantum systems;
- Complete quantum state tomography:
  - ✓ best method for inferring full information about a quantum state;
  - ✗ unfeasible for large systems due to the exponential growth of the number of measurement settings with the size of the system.
- Witness operator: its mean value measured to be less than zero states the presence of entanglement in the system;
- But still many copies of the quantum state are required.

Experimental Detection of Multipartite Entanglement using Witness Operators

Mohamed Bourennane, Manfred Eibl, Christian Kurtsiefer, Sascha Gaertner, Harald Weinfurter, Otfried Gühne, Philipp Hyllus, Dagmar Bruß, Maciej Lewenstein, and Anna Sanpera  
Phys. Rev. Lett. **92**, 087902 – Published 26 February 2004

# The goal

Given a large quantum state, verify whether entanglement is present in it by minimizing time and resources.

- Our approach:

**npj** | Quantum Information

Article | **OPEN** | Published: 15 February 2018

## Single-copy entanglement detection

Aleksandra Dimić & Borivoje Dakić 

*npj Quantum Information* **4**, Article number: 11 (2018) | [Download Citation](#) 

# Simple example: the singlet state

$$|\psi^-\rangle = \frac{|01\rangle - |10\rangle}{\sqrt{2}}$$

- The singlet state shows perfect anti-correlation in all the Pauli bases:

$$\langle X \otimes X \rangle = \langle Y \otimes Y \rangle = \langle Z \otimes Z \rangle = -1$$



$$M_1 = \frac{\mathbb{1} - X \otimes X}{2} \rightarrow 1 \quad \checkmark$$

$$M_2 = \frac{\mathbb{1} - Y \otimes Y}{2} \rightarrow 1 \quad \checkmark$$

$$M_3 = \frac{\mathbb{1} - Z \otimes Z}{2} \rightarrow 1 \quad \checkmark$$

- The probability of success is

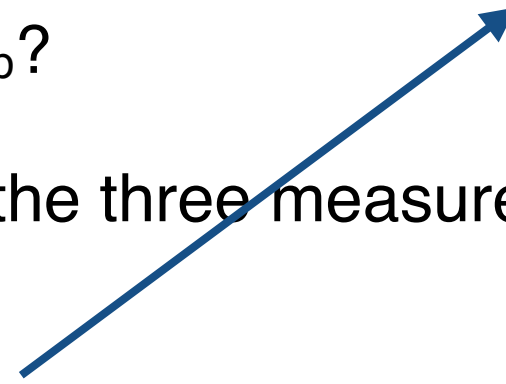
$$P_{|\psi^-\rangle} = \left\langle \frac{M_1 + M_2 + M_3}{3} \right\rangle = 1 = p_e$$



$$\delta = p_e - p_s$$

- And for an arbitrary separable state  $\rho_{\text{sep}}$ ?
- Not possible to obtain success from all the three measurements  $M_1, M_2, M_3$ :

$$P_{\rho_{\text{sep}}} = \left\langle \frac{M_1 + M_2 + M_3}{3} \right\rangle \leq \frac{2}{3} = p_s$$



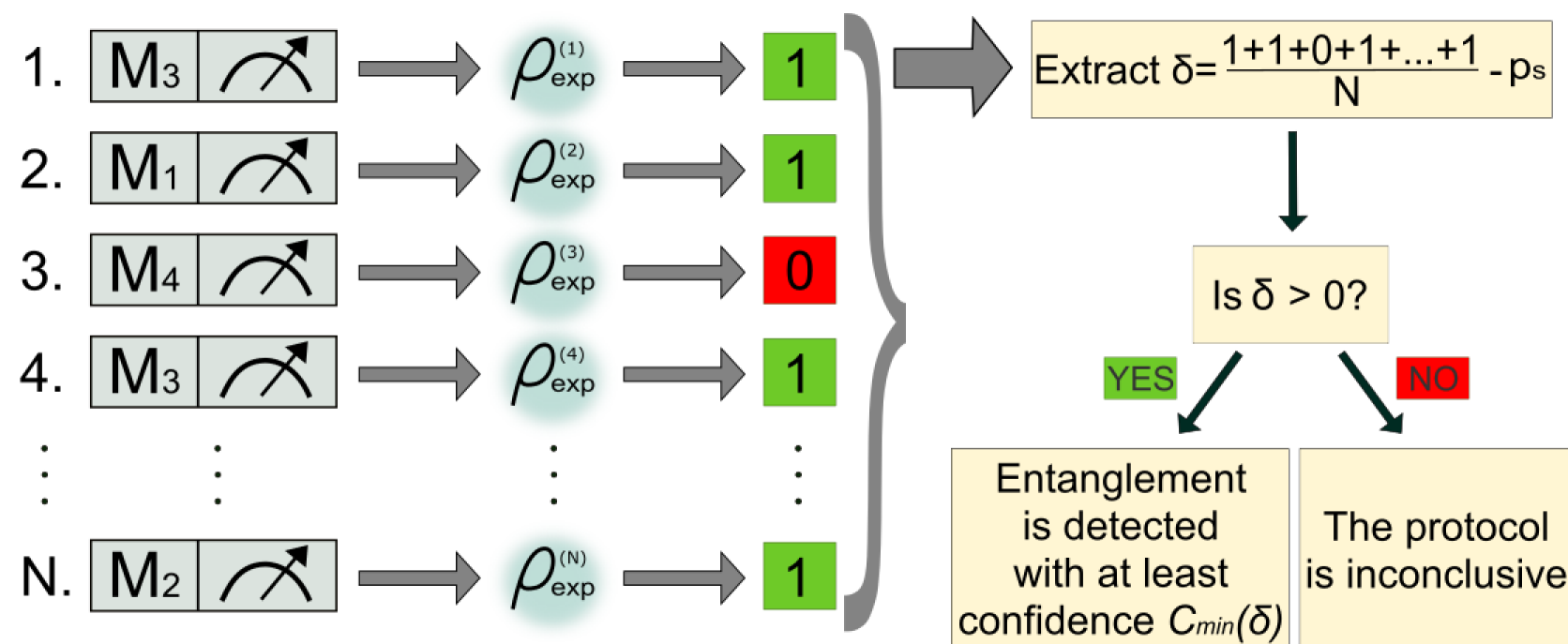
# The general protocol (in experiments)

- Define binary observables  $M_i$  such that they return 1 (success) or 0 (failure);
- Pick them randomly  $N$  times and apply them to the state;
- It is shown<sup>1</sup> that the probability that  $\delta > 0$  for an arbitrary separable state goes exponentially fast to zero with  $N$ :

$$P(\delta) \leq e^{-D(p_s + \delta || p_s)N}$$

- Therefore, the confidence for entanglement detection grows exponentially fast in  $N$ :  

$$C(\delta) = 1 - P(\delta) \geq 1 - e^{-D(p_s + \delta || p_s)N} = C_{\min}(\delta)$$



We recall that...

$$\delta = p_e - p_s$$

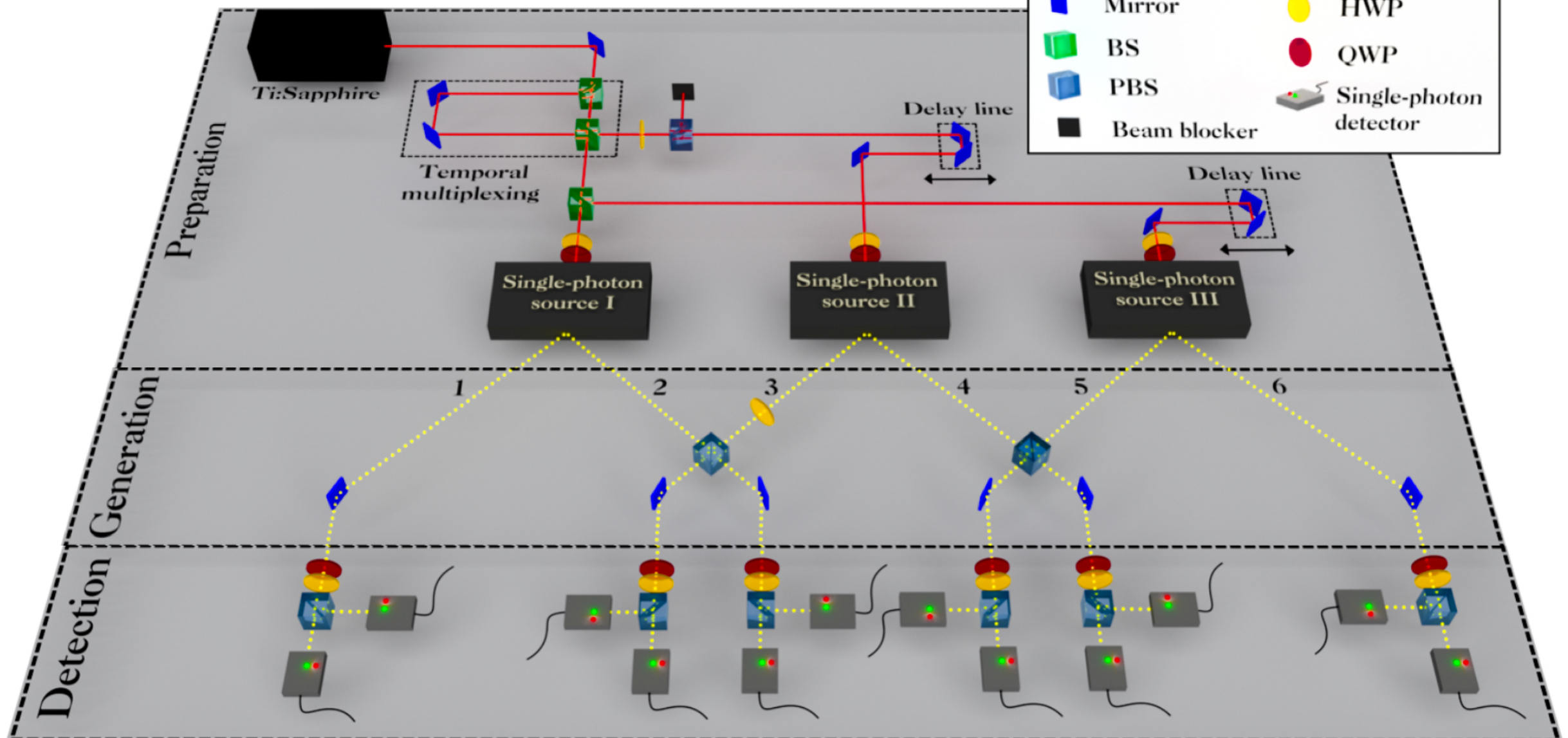
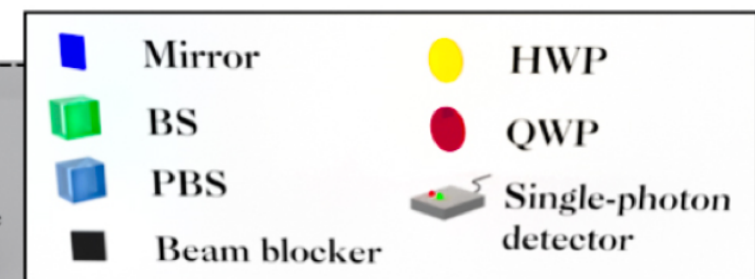
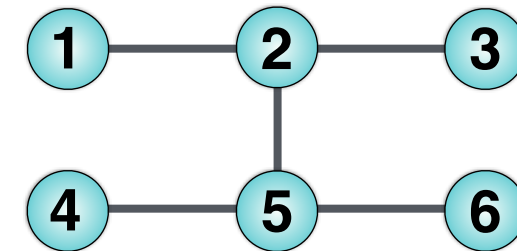
$$P_{|\psi^-\rangle} = \left\langle \frac{M_1 + M_2 + M_3}{3} \right\rangle = 1 = p_e$$

$$P_{\rho_{\text{sep}}} = \left\langle \frac{M_1 + M_2 + M_3}{3} \right\rangle \leq \frac{2}{3} = p_s$$

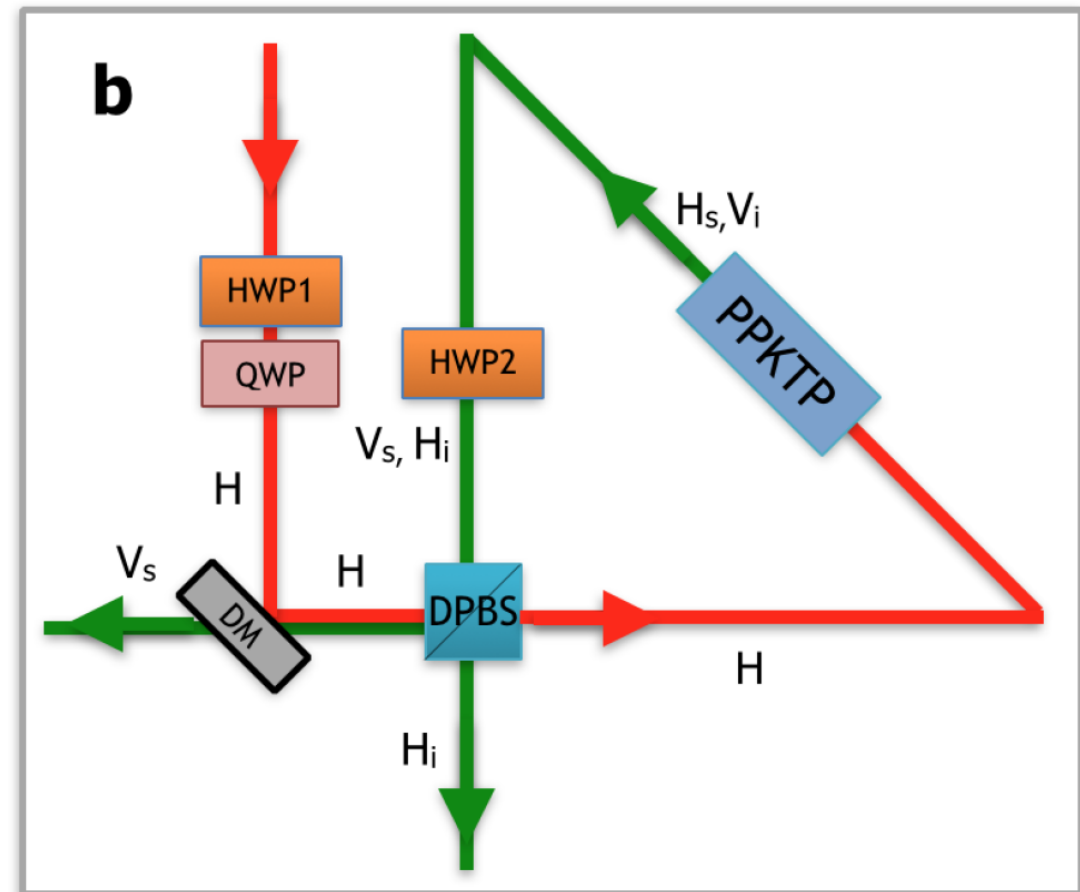
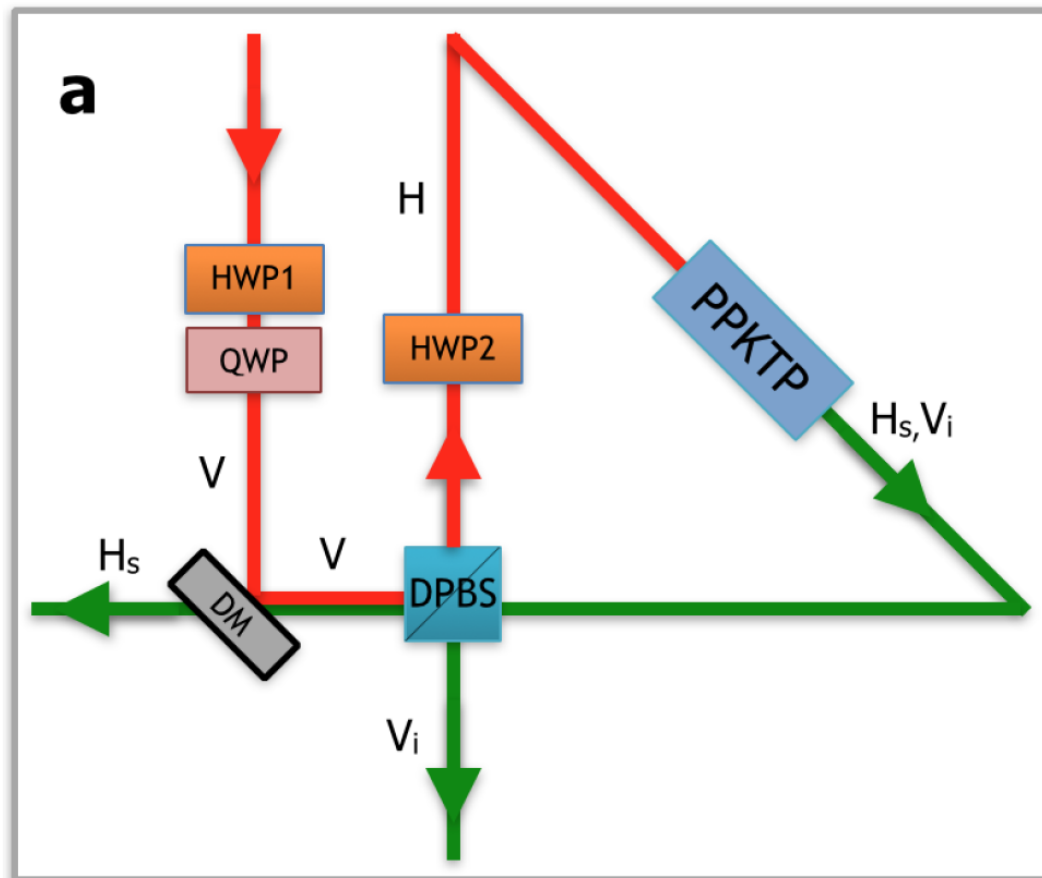
[1] Dimić, A. & Dakić, B. Single-copy entanglement detection. *npj Quantum Information* **4**(1), 11 (2018).

# Six-qubit cluster state generation

$$|Cl_6\rangle = \frac{1}{2}(|H_1H_2H_3H_4H_5H_6\rangle + |H_1H_2H_3V_4V_5V_6\rangle + |V_1V_2V_3H_4H_5H_6\rangle - |V_1V_2V_3V_4V_5V_6\rangle)$$

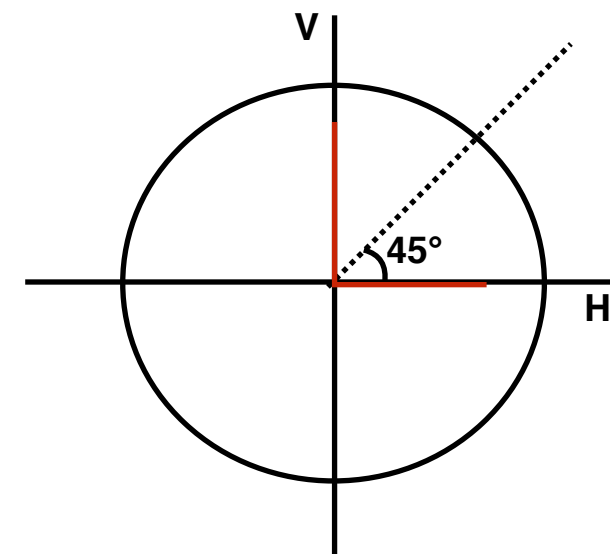


# The single-photon source



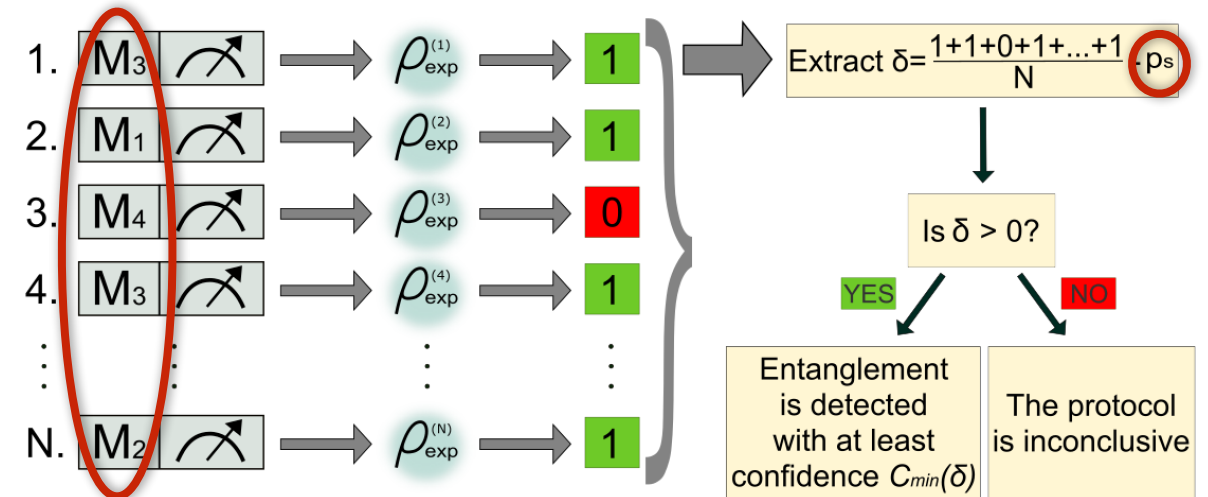
- If the pump is 45 degree polarized, both signal and idler take the same output;

- $$|\psi_{1,2}^-\rangle = \frac{|H_1\rangle|V_2\rangle - |V_1\rangle|H_2\rangle}{\sqrt{2}}$$



# Adaption of the protocol to our state

- It means finding the  $M_i$  operators and the separable bound  $p_s$  for our state;



- We start from two witness operators and translate them into our probabilistic protocol<sup>2</sup>:

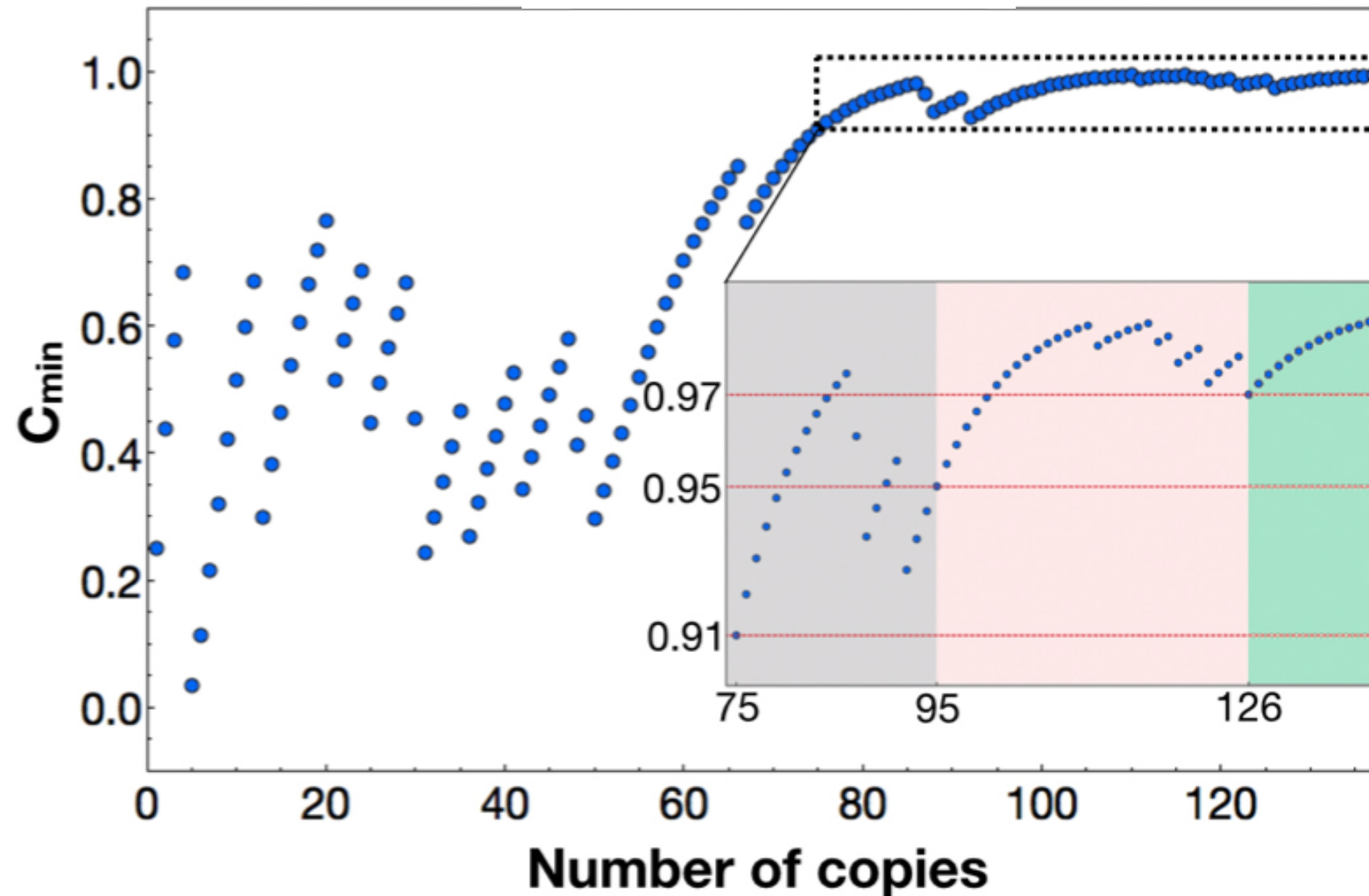
|                                                                                                                                           |                                                                 |
|-------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------|
| $W_1 = 3\mathbb{1} - 2 \left( \prod_{k=1,3,5} \frac{\mathbb{1} + G_k}{2} + \prod_{k=2,4,6} \frac{\mathbb{1} + G_k}{2} \right)$            | $W_2 = \frac{1}{2} \mathbb{1} -  Cl_6\rangle\langle Cl_6 $      |
| $\left\{ M_1 = \prod_{k=1,3,5} \frac{\mathbb{1} + G_k}{2}, \right.$<br>$\left. M_2 = \prod_{k=2,4,6} \frac{\mathbb{1} + G_k}{2} \right\}$ | $M_k = \frac{\mathbb{1} + S_k}{2}$<br>where $k = 1, \dots, 2^6$ |
| $p_{s1} = \frac{3}{4}$                                                                                                                    | $p_{s2} = \frac{3}{4}$                                          |

[2] Tóth, G. & Gühne, O. Detecting genuine multipartite entanglement with two local measurements. *Phys. Rev. Lett.* **94**(6), 060501 (2005).

# Experimental results

2 measurements sampled N=150 times.

$$W_1 = 3\mathbb{1} - 2 \left( \prod_{k=1,3,5} \frac{1 + G_k}{2} + \prod_{k=2,4,6} \frac{1 + G_k}{2} \right)$$



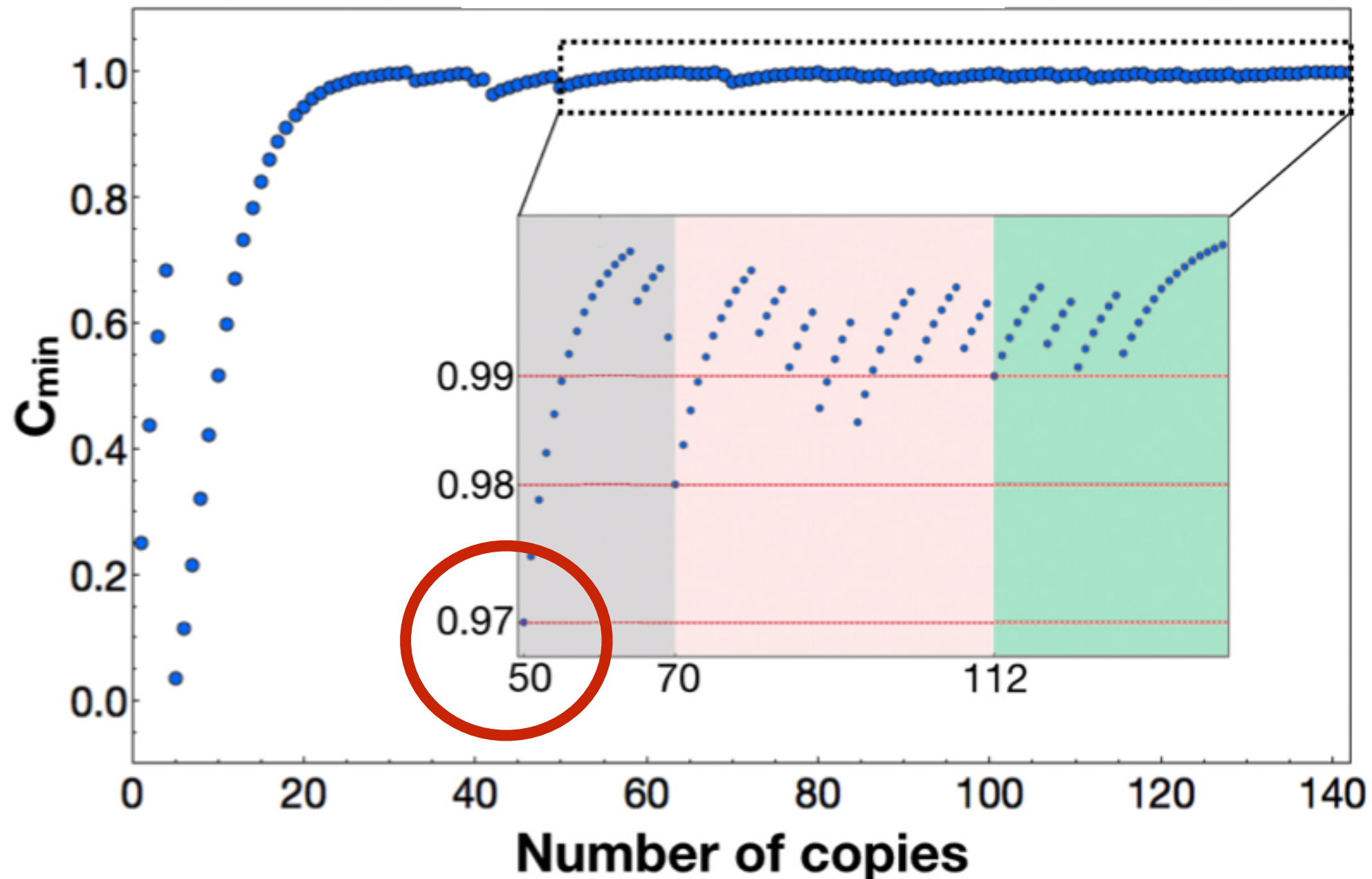
$$C_{\min} = 1 - e^{-D(p_s + \delta || p_s)N}$$

$p_{s1}$

# Experimental results

64 measurements sampled N=160 times.

$$W_2 = \frac{1}{2} \mathbb{1} - |Cl_6\rangle\langle Cl_6|$$



# Conclusions

- We provide a method to detect entanglement with high confidence with a reduced number of copies;
- The protocol is based on a probabilistic procedure and a translation of witness operators into it;
- Rather than measuring mean values of witness operators, we focus on single-copy measurements only.

# The team



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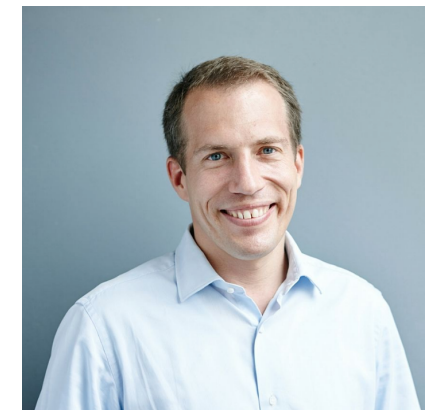


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