A computationally universal phase of quantum matter

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joint work with D.-S. Wang, D.T. Stephen, C. Okay, and H.P. Nautrup
The liquid phase of water
A quantum phase of spins in 2D

... which supports universal quantum computation

We consider:
- Phases of unique ground states of spin Hamiltonians, at $T = 0$,
- In the presence of symmetry,
- In spatial dimension 2 (a lattice of spin 1/2 particles)
A quantum phase of spins in 2D

... which supports universal quantum computation

We show:

- There exists a quantum phase of matter which is universal for quantum computation
- The computational power is *uniform* across the phase.
- Employ measurement-based quantum computation
It can be shown that this process finds a nontrivial divisor of $n$ with probability at least $\frac{1}{2}$, independent of the factors of $n$. A brief sketch of the proof is as follows.

Let $r_i$ be the order of $x \pmod{p_i^{a_i}}$. Consider the largest power of 2 among these powers of 2 agree: if they are all equal and larger than 1, then $x^{r/2}$ is a nontrivial divisor of $n$. By the Chinese remainder theorem, choosing an $x \pmod{n}$ at random, choosing an $x \pmod{p_i^{a_i}}$ at random, where $x \equiv x \pmod{p_i^{a_i}}$ for any choice of $x$, the probability of finding a nontrivial divisor is at least $\frac{1}{2}$. Thus, this process finds a nontrivial divisor of $n$ with probability at least $\frac{1}{2}$.
A lesson from quantum error-correction

... With group and eigenstate, we’ve learned to fix Your quantum errors with our quantum tricks.*

“Good” entanglement often comes with a symmetry

[*] Dan Gottesman, *Quantum error correction sonnet*
Outline

1. Short history of “computational phases of quantum matter”

2. A computationally universal phase of matter in 2D
Part I:

* A short history of

“computational phases of quantum matter”
Measurement-based quantum computation

Unitary transformation

Projective measurement

deterministic, reversible

probabilistic, irreversible
Information written onto the resource state, processed and read out by one-qubit measurements only.

Universal computational resources exist: cluster state, AKLT state.

How rare are MBQC resource states?
1. MBQC resource states are rare

Fraction of useful states smaller than $\exp(-n^2)$

[$n$: number of qubits]

1. MBQC resource states are rare

\[ \text{Fraction of useful states smaller than } \exp(-n^2) \]

\[ n: \text{number of qubits} \]

What about systems with symmetry?

In the presence of symmetry

- Computational power is uniform across physical phases (known in 1D, conjectured beyond).
- Computationally useful quantum states are no longer rare.
Symmetry-protected topological order

Definition of SPT phases:

We consider ground states of Hamiltonians that are invariant under a symmetry group $G$. 
Two points in parameter space lie in the same SPT phase iff they can be connected by a path of Hamiltonians such that

1. At every point on the path, the corresponding Hamiltonian is invariant under $G$.

2. Along the path the energy gap never closes.
2. Symmetry protects computation

we observe low-maintenance features of the ground-code MQC in that this computation is doable without an exact (classical) description of the resource ground state as well as without an initialization to a pure state. It turns out these features are deeply intertwined with the physics of the 1D Haldane phase (cf. Fig. 1), that is well characterized as the symmetry-protected topological order in a modern perspective [6, 7]. We believe our approach must bring the study of MQC, conventionally based on the analysis of the model entangled states (e.g., [1, 8, 9]), much closer to the condensed matter physics, which is aimed to describe characteristic physics based on the Hamiltonian.

3. Symmetry-protected wire in MBQC

- Computational wire persists throughout symmetry-protected phases in 1D.
- Imports group cohomology from the classification of SPT phases.


4. First quantum computational phase

- 1-qubit universal MBQC on a chain of spin-1 particles protected by an $S_4$ symmetry.

5. The role of symmetry breaking

Demonstrate how to measure in symmetry-breaking bases without incurring decoherence.

5. The role of symmetry breaking

- Small amount of symmetry breaking — unitary gates
- Large amount of symmetry breaking — measurement
Hints at the classification of MBQC schemes by symmetry.

Entanglement in the PEPS picture

- The more "bad" entanglement, the harder it is to accumulate rotation angle in the logical gates.
Inspection

The above waypoints 2 - 5 are about 1D systems.

1D is not sufficient for universal MBQC

Here is why:

- MBQC in spatial dimension $D$ maps to the circuit model in dimension $D - 1$

$\Rightarrow$ Require $D \geq 2$ for universality.
Are there computationally universal quantum phases in two dimensions?

This talk describes one.
Part II:

A computationally universal SPT phase in 2D
Description of the 2D phase & result

- The symmetries of the phase are

- The 2D cluster state is inside the phase

Result. For a spin-1/2 lattice on a torus with circumferences $n$ and $Nn$, with $n$ even, all ground states in the 2D cluster phase, except a possible set of measure zero, are universal resources for measurement-based quantum computation on $n/2$ logical qubits.
Consider MBQC resource states as tensor networks
Cluster-like states

... have PEPS tensors with the following symmetries

\[
\begin{align*}
X & \otimes I \\
X & \otimes I \\
Z & \otimes I \\
Z & \otimes I
\end{align*}
\]

The cluster states have the additional symmetry

\[
\begin{align*}
Z & \otimes I \\
X & \otimes I
\end{align*}
\]

(We do not require the latter symmetry for cluster-like states)
Splitting the problem into halves

Part A:

**Lemma 1.** All states in the 2D cluster phase are cluster-like.

Part B:

**Lemma 2.** All cluster-like states, except a set of measure zero, are universal for MBQC.
Part B: Symmetry Lego

Recall the symmetries of cluster-like states:

\[ XZ = XZ = XZ = Z \]

\[ XZ = XZ = XZ = Z \]
B: Cluster-like ⇒ universal

The clock cycle:

• Every byproduct operator is mapped back to itself after $n$ columns ($n =$ circumference).

⇒ If a gate can be done once, it can be done many times.
B: Cluster-like $\Rightarrow$ universal

- Map 2D system to effective 1D system
B: Cluster-like $\Rightarrow$ universal

Universal gate set on $n/2$ qubits
Summary and outlook

- There exists a symmetry-protected phase in 2D with uniform universal computational power for MBQC.

- Symmetry Lego is fun—Try it!

Related: arXiv:1803:00095

Related: arXiv:1806.08780
Lemma 3. [*] Symmetric gapped ground states in the same SPT phase are connected by symmetric local quantum circuits of constant depth.

For any state $|\Phi\rangle$ in the phase,

$$|\Phi\rangle = U_k U_{k-1} \ldots U_1 |2D\text{ cluster}\rangle.$$ 

Look at an individual symmetry-respecting gate in the circuit,

$$U = \sum_j c_j T_j, \text{ with } T_j \in \mathcal{P}.$$ 

Which Pauli observables $T_j$ can be admitted in the expansion?

A: In cluster phase $\Rightarrow$ cluster-like

Which Paulis $T_j$ can be admitted in the expansion $U = \sum_j c_j T_j$?
A: In cluster phase $\Rightarrow$ cluster-like

Which Paulis $T_j$ can be admitted in the expansion $U = \sum_j c_j T_j$?

Only $X$-type Pauli operators survive in the expansion.
A: In cluster phase $\Rightarrow$ cluster-like

- Local tensors $A_\Phi$ describing $|\Phi\rangle$ are invariant under the cluster-like symmetries.
The parameter $\nu$

There is a complex-valued parameter $\nu$, $|\nu| \leq 1$, that needs to be known about the location of the resource state within the phase.

For infinitesimal angles $d\beta$, this results in a logical rotation [\*]

$$e^{i d\beta |\nu| T},$$

for some Pauli operator $T$. (E.g., $T = Z_k, X_k, X_{k-1}Z_kX_{k+1}$).

We require that $\nu \neq 0$.